Vertically Integrated North–South Trade and the Redux Model

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Abstract
The paper uses the framework of Obstfeld and Rogoff’s Redux model to study the impact of monetary shocks on exchange rate, terms of trade, and welfare in the context of a North–South trade. The authors show that a relative Northern monetary expansion can depreciate or appreciate its currency depending on whether the consumption elasticity of money demand and the degree of monopolistic distortion are low or high enough. This shock has asymmetric effects on welfare in such a way that “beggar-thyself” or “beggar-thy-neighbor” effects always occur.

1. Introduction
It is widely acknowledged that in the last several decades world trade has increased drastically. In this sense, many works have linked these high growth rates to causes such as both lower tariffs and transportation costs as well as income growth. Nevertheless, other recent empirical research also seems to document the important role played by the new production process structure characterized by the location of steps in the production chain in different countries. This phenomenon, so-called vertical specialization, has become increasingly important over the last 20–30 years, accounting for one-third of the growth in trade. Although the geographic orientation of this share of trade is primarily North–North, there is also a significant vertical link North–South. An example is the US pattern, which has become more oriented toward developing countries at the same time that vertical specialization has grown rapidly. More specifically, between 1970 and 1990, the share of total US vertical specialization consisting of imports originating from the South rose from 26% to 37%. In this paper, we focus on this later geographic orientation of vertical specialization by constructing a model of North–South intraindustry trade in intermediates for finished manufactures (“vertical” exchange).

A common thread underlying the so-called North–South models is that the world economy system is divided into a “center” and a “periphery” which are characterized by fundamental asymmetries in the structure and performance of their economies. The center is formed by the advanced countries as a whole, the “North,” and the periphery by those less developed countries as a whole, the “South.” In this paper we will assume that the asymmetry consists of the South as a final-goods (“manufacturer”) producer–consumer and the North as an intermediate-goods (e.g., fabrics, electronic...
components) producer and a final-goods consumer.\textsuperscript{6} Hence, the pattern of North–South trade considered in this paper incorporates the idea of vertical specialization by assuming that the North exports intermediate goods to the South, where they are used to produce final goods which are bound for worldwide consumption.\textsuperscript{7} As was noted earlier, this pattern of trade is linked to relevant empirical features, since many of the key manufactured exports of developing countries involve the importation of intermediate manufactured goods from developed countries, which are then assembled domestically and reexported to world markets.

This paper, applying the \textit{Redux} model developed by Obstfeld and Rogoff (1995), investigates the influence of vertical specialization in a North–South trade model on the macroeconomics and welfare effects derived from relative monetary shocks. Obstfeld and Rogoff’s work launched the research program on “the new open economy macroeconomics” whose main feature is the introduction of nominal rigidities and market imperfections into an intertemporal approach with explicit microfoundations. Moreover, the framework introduced by the \textit{Redux} model allows for a more solid welfare analysis by using explicit utility functions for the representative agents. Thus, in this framework we will assume a worldwide economy characterized by the fact of that while Northern labor is used to produce (intermediate) goods, the South (developing countries) is essentially selling labor services used in assembly operations of final goods, rather than selling the entire product.

Intermediate products have been introduced in earlier papers in the context of dynamic general-equilibrium models with optimizing monopolistic, competitive price setters, but in a different way. Thus, Chari et al. (1998) consider home final goods as having been produced from intermediate goods produced in the home and foreign countries. They find that, in general, a monetary shock does not generate persistent output responses. Bergin and Feenstra (1999) and (2000) use a particular production structure proposed by Basu (1995) with intermediate goods together with translog preferences in order to explain endogenous persistence in a staggering framework.

Throughout this paper, it is shown that vertical specialization plays a key role in the explanation of exchange rate movements as well as in the international monetary transmission mechanism. Thus, without vertical specialization, traditional “expenditure-switching” effects operate by inducing a shift in world demand toward goods from the country whose currency is weakening. In this framework, monetary disturbances tend to bring about a low or even negative cross-country output correlation and international comovements in consumption, which seems to be counterfactual.\textsuperscript{8} With vertical specialization, however, by eliminating the expenditure-switching effects of exchange rate movements, that ordering is reversed, which is more in line with the evidence on international business cycles. Furthermore, working through the cross-country input–output relations, our model explains persistent effects of monetary shocks on worldwide variables.

On the other hand, although—as in the \textit{Redux} model—the no-overshooting result characterizes our model, other different and interesting results emerge with vertical specialization. First, the effect of a monetary shock on the exchange rate is potentially ambiguous; that is, the Northern currency could depreciate or appreciate in response to a relative Northern monetary expansion. This occurs because demand changes for both final and intermediate goods induced by this monetary shock work in opposite directions on demand for Northern currency. The net effect depends on structural parameters of economies such as the consumption elasticity of money demand and the degree of monopoly power that producers have.
Second, even when—as in the Redux model—the increase in the relative money supply in the North gives rise to a depreciation of its currency, we find much greater exchange rate volatility, compared with a situation without vertical integration in trade. The reason is that, under vertical specialization, if the North currency depreciates, the relative Northern consumption must decline, requiring accordingly, from the money market equilibrium, a magnified response in the exchange rate.

Third, an important implication of the earlier results is that, contrary to the Redux model again, a relative monetary shock always generates asymmetric impacts on Northern and Southern welfare. More specifically, we find that a relative Northern monetary expansion leads to a “beggar-thy-neighbor” phenomenon when the Northern currency appreciates, but adversely affects the North, the “beggar-thyself” problem, when the shock brings about a depreciation of its currency. In this result a key role is played by the real revenue-switching effect associated with the terms-of-trade adjustment, which, in turn, is linked to exchange rate movements.

The Obstfeld–Rogoff framework has been developed in some other directions by relaxing some of its assumptions. Here we highlight only those contributions whose results are closest to that of our setup.9 Betts and Devereux (2000) have introduced the assumption of “pricing-to-market”—that is, when some firms segment markets by country and set prices in the local currency of the buyer. They find that pricing to market in combination with local-currency sticky prices (and reduced passthrough of exchange-rate changes to prices) increase the volatility of exchange rate, leading, unlike in our model, to a possible “overshooting” of the nominal exchange rate. They also show that, as in our model, their setup increases the possibility of greater comovements in home and foreign outputs than in consumption. Finally, they obtain, in contrast to our setup now, that monetary policy is always a “beggar-thy-neighbor” instrument.

Two further extensions of the Redux model can be found in Corsetti and Pesenti (2001) and Tille (2001), who relax the Redux model assumption that the elasticity of substitution between two goods produced in the same country is the same as the elasticity of substitution between two goods produced in different countries. Corsetti and Pesenti (2001) assume that the elasticity across countries is equal to unity, being hence lower than the elasticity within each country, which is greater than one, as in Obstfeld–Rogoff. Although their analysis presents the analytical advantage of closed-form solutions (making it possible to analyze the impact of large shocks), it does not allow for dynamics effects through the current account. On the other hand, Tille (2001) also extends Obstfeld–Rogoff by allowing for different elasticities of substitution across and within countries. Moreover, he extends Corsetti–Pesenti by allowing for current account effects as well as by generalizing the treatment of the differences of elasticity. A central result of his paper is that a low substitutability across countries is a source of exchange rate volatility and can lead to a “beggar-thyself” effect in the country where a shock takes place.

While the results of our model are quite different from those of the Corsetti–Pesenti model (except that the no-overshooting result also characterizes this model), they do, however, present a certain formal similarity to those of Tille in the extreme case of non-substitutability between home and foreign goods. But, even in that case, there are two substantial differences between our framework and that of Tille. First, while Tille’s results would be linked to a very extreme case of agents’ preferences, our findings are determined by the connection of economies in a vertical trading chain. Second, as a consequence of such a trade structure, a money shock now affects not only worldwide variables (consumption and output) in the short run, but also in the long run; that is,
its effects on the world economy are persistent. This is due, in contrast to the other extensions, to the use of tradable intermediates in the production function of final goods, which generates a variable markup on the effort of producing them as the exchange rate changes.

We will hereafter identify the “North” as home country and the “South” as foreign country.

Section 2 describes the model and the equilibrium conditions. Section 3 discusses the steady-state conditions, and section 4 does the same for the short run and long run once it is given an exogenous monetary shock. In section 5, we examine the effects of the changes we consider on welfare. Section 6 provides a brief numerical illustration, and section 7 concludes.

2. The Model

Consider a two-country global economy model which is structured around that of the Redux model by Obstfeld and Rogoff (1995). Thus, we assume that the world is populated by a continuum of agents on the interval [0, 1] and consists of two countries (home and foreign) of identical size. The pattern of trade is characterized by vertical integration. Agents of the home country, on the interval [0, \(\frac{1}{2}\)], produce differentiated intermediate goods which are then assembled by agents of the foreign country, on the interval \((\frac{1}{2}, 1]\), and reexported to world markets as final goods.

Using \(\bar{C}(z^*)\) to denote a home individual’s consumption of final good \(z^*\), we define the domestic consumption index by

\[
C = \frac{1}{(\frac{1}{2})^{(1/\theta^*-1)}} \int_{\frac{1}{2}}^{1} \bar{C}(z^*)^{(1/\theta^*-1)/\theta^*} dz^*^{\theta^*/(\theta^*-1)}.
\]

(1)

The home price index corresponding to equation (1) is

\[
P = \frac{1}{(\frac{1}{2})^{(1/\theta^*-1)}} \int_{\frac{1}{2}}^{1} P(z^*)^{1-\theta^*} dz^*^{1/(1-\theta^*)},
\]

(2)

where \(P(z^*)\) is the home price of final good \(z^*\) and \(\theta^* > 1\) is the elasticity of substitution between two different final goods. The foreign consumption index is defined analogously, with asterisks denoting foreign variables.

Following the Redux model and some of its extensions, for instance Tille (2001), we consider that production of intermediate goods is undertaken by home households, each of which is the sole producer of a differentiated good.

We assume that each representative foreign agent produces (assembles) a final brand \(z^*\) by using his/her own labor and imported intermediates. As is fairly standard for simple treatments of intermediates, we consider that the foreign technology is Leontief, so the foreign production function is

\[
Y^*(z^*) = \min [AL^*(z^*), DI^*(z^*)],
\]

(3)

where \(L^*(z^*)\) is the foreign (Southern) labor used, and \(I^*(z^*)\) is the quantity of the intermediate index used to produce final goods given by

\[
I^*(z^*) = \frac{1}{(\frac{1}{2})^{(1/\theta-1)}} \left( \int_{0}^{\frac{1}{2}} I^*(z)^{\theta-1}/\theta \ dz \right)^{\theta/(\theta-1)}.
\]

(4)
Also, $\vartheta > 1$ denotes the elasticity of substitution between two varieties of intermediate goods. In (3), $A$ and $D$ are positive constants. The key dimension is the role of intermediate inputs (labor adjusts as a residual); in particular, $0 < D < 1$ represents the amount of output produced by a unit of intermediate.\(^\text{12}\) In consequence, $1/D$ is the amount of intermediate required to produce one unit of output of final good $z^\star$.

Moreover, as in the Redux model, the parameters $\vartheta$ and $\vartheta^\star$ are a (decreasing) index of imperfect competition or a (decreasing) index of markup due to monopolistic distortion. The values of $\vartheta$ and $\vartheta^\star$ can differ or be equal, but in order to simplify the exposition of the model we will assume that $\vartheta = \vartheta^\star$. In any case, the sign of $\vartheta - \vartheta^\star$ is unclear. Thus, the implicit assumption in the Bils (1987) and Rotemberg and Woodford (1999) frameworks seems to be that final goods are sold under conditions of monopoly power, whereas inputs (such as raw materials) are sold under competitive conditions. However, as Barro and Tenreyro (2000) have pointed out, from an empirical standpoint, it is not obvious that this assumption is reasonable, especially if we think of inputs as intermediate manufactured goods (for instance, electronic components or fabrics).

Analogously to equation (2), the domestic intermediate price index corresponding to equation (4) is

$$P_I = \frac{1}{(1)^{1/(1-\vartheta)}} \left[ \int_0^1 P_I(z)^{1-\vartheta} \, dz \right]^{1/(1-\vartheta)},$$

where $P_I(z)$ represents the home currency price of the intermediate good $z$.

We now consider the home agent’s intertemporal optimization problem using a utility function similar to that of the Redux model

$$U_i = \sum_{s=0}^{\infty} \beta^{-s} \left[ \log C_s + \frac{\chi}{1-\varepsilon} \left( \frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{k}{2} Y_s(z)^2 \right],$$

where $0 < \beta < 1$ is the discount rate, and $\chi, \varepsilon, k$ are positive parameters. $M_s$ represents nominal money balances held by the home representative agent at time $s$, and $C_s$ is a consumption index defined above. The final term in the expression in square brackets in (6) captures the disutility (work effort) to the home agent of producing output $Y_s$ of a particular brand of intermediates; it combines the relationship between output and the producer’s effort (labor input) used to produce the output with that between effort and disutility.

The home individual’s demand for final good $z^\star$ resulting from cost minimization is given by $C(z^\star) = [P(z^\star)/P]^{-\vartheta} 2C$. Then each foreign consumer–producer faces the constant-elasticity demand $\vartheta$ given by

$$Y^d(z^\star) = [P^\star(z^\star)/P^\star]^{-\vartheta} 2C^W,$$

$C^W$ being the aggregate global consumption. Similarly, using production function (3) we may write the demand curve which each home producer faces for his/her output as

$$Y^d(z) = [P_I(z)/P_I]^{-\vartheta} Y^\star(z^\star)/D.$$

The domestic budget constraint in each period is

$$P_iB_i + M_i = P_i(1 + r_{i-1})B_{i-1} + M_{i-1} + P_{I,i}(z)Y_i(z) - P_iC_i - P_iT_i,$$
where \( r \) is the real interest rate of a real bond \((B)\), denominated in the composite consumption good, and \( T \) represents the real taxes paid to the domestic government.\(^{13}\)

As is standard in models of this type, starting with Blanchard and Kiyotaki (1987), we assume that all producers in each country are symmetric, which implies that they charge the same price in equilibrium. Moreover it is assumed that there are no impediments or costs to trade between the countries and the law of one price is verified.

The foreign representative agent maximizes a utility function that is similar to that of the home representative agent, but the dynamic budget constraint is now

\[
P_i^* B_i^* + M_i^* = P_i^*(1 + r_{i-1}) B_{i-1}^* + M_{i-1}^* + P_i^*(z^*) Y_i^*(z^*) - P_i^* Y_i^*(z^*)/ D - P_i^* C_i^* - P_i^* T_i^*.
\]

Equation (10) incorporates a new ingredient: the input–output connections across countries. Thus, the term \( P_i^* Y_i^*(z^*)/ D \) represents the intermediate-good costs employed in the production process of final goods.

From the appropriate Lagrangean (not shown), we derive the following first-order conditions for maximization:

\[
C_{t+1} = \beta(1 + \eta_i) C_i; \quad C_i^* = \beta(1 + \eta_i) C_i^*; \quad (11)
\]

\[
\frac{M_i}{P_i} = \left[ \chi C_i \left( \frac{1 + \eta_i}{i_t} \right) \right]^{1/\epsilon}; \quad \frac{M_i^*}{P_i^*} = \left[ \chi C_i^* \left( \frac{1 + \eta_i}{i_t^*} \right) \right]^{1/\epsilon}; \quad (12)
\]

\[
k Y_i(z) = \frac{1}{P_i C_i} \left[ \frac{1}{\theta} - 1 \right] P_i Y_i(z) \left[ Y_i^*(z^*) \right]^{1/\theta}; \quad (13)
\]

\[
k Y_i^*(z^*) = \frac{1}{P_i^* C_i^*} \left[ \frac{1}{\theta} - 1 \right] P_i^* Y_i^*(z^*) \left[ 2 C_i^* \right]^{1/\theta} \left[ \frac{P_i^*}{D} - \frac{P_i^*}{D} \right]. \quad (14)
\]

Equations (11) represent the standard Euler conditions for the optimal inter-temporal allocation of consumption, and equations (12) are the money demand for market-clearing conditions. Both equations (13) and (14) equate the marginal utility of the additional net revenue from producing an extra unit of output to its marginal disutility of the needed effort. Thus, equation (13) shows that an increase in the price of intermediate goods raises revenue leading to a greater marginal effort and, as a consequence, a greater output. In turn, equation (14) says that an increase in the price of intermediate goods, \textit{ceteris paribus}, reduces the marginal revenue, leading thereby to both marginal effort and output reduction.

### 3. Steady-state Conditions

We start by deriving the solution of the model for an initial steady-state situation in which prices are fully flexible and the money supply is constant. If we denote the steady state’s variables by an upper bar, in this framework the individual consumption levels are subject to the following conditions:

\[
\bar{C} = \bar{r} \bar{B} + \bar{P}_i(z) \bar{Y}(z) \quad \bar{C}_i^* = -\bar{r} \bar{B} + \left[ \bar{P}_i(z^*) - \frac{\bar{P}_i}{D} \right] \bar{Y}_i^*(z^*) \bar{P}_i^*.
\]

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Equations (15) are the long-run current account conditions. The first says that, in the initial steady state, per capita home consumption is equal to net interest payments from the rest of the world plus real value of output. The second has a similar interpretation for the foreign country with the exception of considering output net of intermediate spending (output minus intermediate spending, both measured in units of the composite consumption good).

We will denote by subscripts “0” the values of the variables in the initial steady state. Moreover, following the Redux model, we will assume, at the starting point, the particular case in which stocks of bonds are zero, \( B_0 = B^*_0 = 0 \) (that is, no country has any net claims on the other). Then, using (12)–(15), we obtain the following results:

\[
\begin{align*}
\overline{Y}_0 &= \left( \frac{\vartheta - 1}{\vartheta k} \right)^{1/2}; \quad \overline{Y}_0^* = D \left( \frac{\vartheta - 1}{\vartheta k} \right)^{1/2}; \\
\frac{\overline{M}_0}{\overline{P}_0} &= \left( \frac{\chi S_0 Y_0}{1 - \beta} \right)^{1/e}; \quad \frac{\overline{M}^*_0}{\overline{P}^*_0} = \left( \frac{\chi Y^*_0}{\vartheta (1 - k Y^*_0)(1 - \beta)} \right)^{1/e}; \\
\overline{S}_0 &= D(\vartheta - 1)(1 - D^2)/[\vartheta - D^2(\vartheta - 1)];
\end{align*}
\]

where, in (18), \( \overline{S}_0 \) represents the terms of trade of the home country in the initial steady state (\( \overline{P}_0/\overline{P}^*_0 \)). Equations (16) show that both countries’ steady-state production depends on the degree of competition in the goods markets. The more output is pushed below its competitive levels, the lower is the value of parameter \( \vartheta \) (that is, the lower is the degree of competition in the markets). In fact, \( \vartheta \) determines a markup equal to \( \vartheta (\vartheta - 1)^{-1} \). In addition, since \( D \) is assumed to be lower than one, the above equations show that final goods output is a constant share of intermediate goods output. Finally, equations (17) reveal that the assumption of vertically integrated trade introduces—unlike in the Redux model—an asymmetry in the relevant conditions for optimization of real money holdings in the initial steady state.

We next present the analysis in terms of log-linear approximations around the initial steady state. We denote by lower-case letters the percentage deviations from the initial steady state. That is, for any variable \( X \), \( x \) represents \( (X - \overline{X}_0)/\overline{X}_0 \). The only exception is that we will represent as \( b = (B - \overline{B}_0)/\overline{C}_0^W \).

By assuming that purchasing power parity holds and the fact that agents are symmetric within a country, from (7) and (8) we may write

\[
y = y^* = c^W.
\]

This equation can be interpreted as the world demand schedule for final and intermediate goods. Also, (19) reflects the main feature of our setup: the assumption of vertically integrated international trade links international outputs and induces a positive correlation between them.

To solve the model, we still need to obtain the change in the steady state for domestic and foreign consumption as well as for the terms of trade. So we will denote these changes in the steady state by an upper bar on the lower-case letters; that is, \( \bar{x} = (\overline{x} - \overline{x}_0)/\overline{x}_0 \). Thus, after some algebraical manipulation (given in the Appendix) we express the solutions for consumption and terms of trade as a function of foreign assets:
Equations (20) and (21) come from goods market clearing and balance-of-payments equilibrium conditions. Thus, for instance, in (20) we can see that a home country trade surplus in response to the monetary shock, implying $b > 0$, will permanently raise (decrease) domestic (foreign) consumption. On the other hand, (21) shows a characteristic of the models with infinitely living agents in both countries and equal constant discount rates in which an international transfer of assets leads to a permanent change in the terms of trade.

4. Short-run and Long-run Conditions: the Monetary Shock

In this paper we are only interested in the analysis of the effects of monetary shocks. To this end we will consider that at time $t$ the economy is affected by an unanticipated permanent monetary shock ($m = \bar{m}$; $m^* = \bar{m}^*$). As in other models of this type, we will analyze these effects in two different scenarios: in the short run, in which prices are set for period $t$, and in the long run, from period $t + 1$ on, where the economy comes into a new steady state and prices can be adjusted.

As, in the short run, the economy can run a current-account imbalance, we make use of the intertemporal budget constraints in order to obtain changes in net foreign assets. At the same time, since, in the short run, output becomes demand-determined when prices are sticky, equations (13) and (14) do not bind here. Hence, taking this into account, from (9) and (10), we derive after some manipulation the following current account equation, details of which are given in the Appendix:

$$\bar{b} = -\frac{2\bar{S}_0}{D}\left[\left(1 - \frac{\bar{S}_0}{D}\right)(c - c^e) + e\right].$$ (22)

To see the intuition behind (22), for instance, hold $\beta$ constant and ask what effect an exchange rate change would have on home consumption relative to foreign consumption in a world where the economies are vertically integrated. In such a scenario, while relative outputs remain unaltered, relative incomes are affected. Take, for instance, the case of an exchange rate appreciation. With vertical specialization, an appreciation raises the home currency earnings of home agents, and reduces the foreign currency earnings of foreign agents, at given relative production levels. Hence, an appreciation generates a world redistribution of income towards the home country, which raises home consumption relative to foreign consumption. Instead, without vertical specialization, an appreciation would cause world demand shifts towards the foreign country, leading to a decrease in production, income, and accordingly consumption in the home country relative to the foreign country. Therefore, with vertical specialization, since there is no expenditure-switching effect, the only role of exchange rate movements is in a wealth (income) effect via a change in the terms of trade and a transfer of income from one country to the other.
From (22), and by making use of (20) which explains the behavior of home and foreign consumption in the long run, as well as the fact that relative consumption changes are permanent \((c - c^* = \bar{c} - \bar{c}^*)\), we arrive at a first schedule in \(e\) and \(c - c^*\) (the so-called GG schedule in the Redux model) given by

\[
e = -\frac{1}{\vartheta - D^2(\vartheta - 1)} \left[ \frac{1 - D^2}{\bar{r}} + 1 \right] (c - c^*). \tag{23}
\]

Equation (23) shows the home currency appreciation needed to raise relative home income enough to finance a given permanent rise in relative home consumption. Contrary to the Redux model, the slope of the GG schedule is negative. The reason is implicit in the aforementioned intuition behind (22): since, as is shown in equation (19), the relative output remains unaltered, an appreciation induces a wealth effect in the home agents because it makes imported final goods cheaper. In turn, higher incomes translate into higher relative consumption. That said, the higher (lower) the degree of monopolistic distortion—the lower (greater) the value of elasticity \(J\)—the lower (greater) is the response of relative consumption changes to exchange rate movements. This is because, with nominal prices fixed in the short run, the more competitive good markets become, the larger is the rise in relative home income derived from an appreciation of the home currency.\(^{14}\)

On the other hand, in the money market, we have a similar framework to that in the Redux model, the so-called MM schedule.\(^{15}\) So, after a permanent unanticipated relative monetary shock, from (12) we find the relationship between relative consumption changes and the exchange rate movements which clear the money market:

\[
e = (\bar{m} - \bar{m}^*) - e^{-1}(c - c^*). \tag{24}
\]

Equation (24) shows, for a given permanent relative monetary shock, how relative consumption changes, by changing relative money demand, requiring opposite-sign movements in exchange rates to clear the money market.

Using (23) and (24) we obtain the impact of the monetary shock on exchange rate and relative consumption change, given by equations (25) and (26), respectively:

\[
e = \frac{\varepsilon(1 - D^2 + \bar{r})}{\varepsilon(1 - D^2 + \bar{r}) - \bar{r}[\vartheta - D^2(\vartheta - 1)]} (\bar{m} - \bar{m}^*), \tag{25}
\]

\[
c - c^* = -\frac{\varepsilon \vartheta[\vartheta - D^2(\vartheta - 1)]}{\varepsilon(1 - D^2 + \bar{r}) - \bar{r}[\vartheta - D^2(\vartheta - 1)]} (\bar{m} - \bar{m}^*). \tag{26}
\]

Figure 1 shows these graphical solutions for the exchange rates and the relative consumption changes under the Redux model’s diagram.

As is shown in equation (24), the money-market equilibrium requires that a permanent relative monetary expansion in the home country (in Figure 1 this corresponds to an upward shift of the initial MM schedule to \(M'M'\)) be accommodated by relative consumption or exchange rate changes. In graphical terms, this depends on the size of the slope of GG and MM schedules, which determine, other things being equal, how a monetary disturbance translates to relative consumption or exchange rate changes as well as the direction of those changes.
This is reflected in equations (25) and (26) in which, in contrast to the Redux model, the effect of relative monetary expansion on exchange rate and relative consumption is ambiguous. Thus, a permanent relative home monetary expansion can give rise to a depreciation or an appreciation of the home currency (points A or B, respectively, in Figure 1), depending on the value of the structural parameters of that economy, such as $e$ and $\vartheta$. Specifically, from (25), we derive that

\[(27)\]

In (27) we see that the effect on exchange rate, other things being equal, depends on the interplay between the degree of monopolist distortion (linked to the values of $\vartheta$) and the consumption elasticity of money demand ($1/e$). The former refers to the sensitivity of relative consumption changes to price changes (exchange rate, in the short run) via accumulation or disaccumulation of net foreign assets. The latter is the degree to which consumption changes alter the money demand. Condition (27) states that by combining low degree of monopolist distortion with high consumption elasticity of money demand (high values of $\vartheta$ and low values of $e$) it is possible to find that the home currency appreciates in response to a relative home monetary expansion. Intuitively this can be explained as follows.

In such a situation, suppose that there were not an appreciation, but a depreciation ($e > 0$). With nominal prices fixed in the short run and high values of $\vartheta$, a depreciation of the home currency would lead to a large decline in relative domestic income, and hence in the relative home consumption. At the same time, taking into account the high value of $1/e$, it would in turn lead to a proportionally greater fall in relative home money demand, making it impossible to reach thereby the money market equilibrium. Therefore, at such high values of $\vartheta$ and $1/e$, a rise in home relative money supply would be consistent with money market equilibrium conditions only under an appreciation of the home currency, and hence a rise in the relative home consumption (point B in Figure 1). On the other hand, if we combine a high enough monopolist distortion with a low enough consumption elasticity of money demand (low values of $\vartheta$ and high values of $e$), the home currency tends to depreciate (point A in Figure 1). The reason

\[\text{Figure 1. A Permanent Unanticipated Relative Home Money Supply Increase}\]
is that, under this parameter combination, the impact on consumption and money demand is limited and now an appreciation would lead to a small increase in the relative real income and consumption. Hence, for a low enough consumption elasticity of money demand, the impact of a small rise in relative home consumption on the relative money demand cannot be high enough to clear the money market. In a scenario of vertical specialization, we may conclude that the greater the degree of monopolistic distortion and the lower the elasticity of money demand, the greater is the possibility of a depreciation of the home currency. In addition, in that case, the increase in the exchange rate is greater than the increase in relative home money supply. This is because the rise in the exchange rate is consistent only with a decline in \( \frac{C}{C^*} \), and hence the size of the exchange rate movement required to satisfy the monetary equilibrium condition is enlarged.

Another interesting feature of our setup is that the ingredient of a vertical trading chain plays an important role also in propagating home monetary shocks to the world economy in the long run. This can be seen by solving the model to obtain the long-run behavior of outputs and consumption as well as the impact on current account, which are given by

\[
\tilde{\epsilon}^W = \tilde{y} = \tilde{y}^* = \frac{\epsilon \varphi (1 - D^2)}{2\{\epsilon (1 - D^2 + \overline{\varphi}) - \overline{\varphi}[\vartheta - D^2(\vartheta - 1)]\}(\overline{m} - \overline{m}^*)}, \tag{28}
\]

\[
\overline{b} = -\frac{2\epsilon(\vartheta - 1)(1 - D^2)}{[\vartheta - D^2(\vartheta - 1)]\{\epsilon (1 - D^2 + \overline{\varphi}) - \overline{\varphi}[\vartheta - D^2(\vartheta - 1)]\}(\overline{m} - \overline{m}^*)}. \tag{29}
\]

In the absence of intermediate goods, the optimal final good production is obtained with a “markup” over the marginal rate of substitution between leisure and consumption. With intermediate inputs, however, its price level enters marginal cost as an additional component, which alters the variability of marginal cost and hence optimal output and consumption. Thus, for instance, if the home currency depreciates in response to a relative home monetary expansion, it lowers the effective price of intermediates exported to the foreign country (in foreign currency units). As we can see in equation (14), other things being equal, this tends to raise the marginal utility of the additional revenue earned from producing an extra unit of final good, leading foreign producers to raise their effort and output. Meanwhile, the worsening of the home terms of trade generates a home current account deficit, as is illustrated by equation (29), which reduces long-run home consumption and consequently increases optimal output. Therefore—in contrast to the Redux model again—a depreciation in the home currency, following a surprise monetary expansion in the home economy, will raise both worldwide output and consumption in the long run. In a similar way, one could explain that the long-run effect on both worldwide output and consumption would be reversed when the home monetary shock involves an appreciation of the home currency.

5. Welfare Analysis

One most attractive aspect of the Redux model’s approach is to provide an analytical framework where it is possible to analyze the welfare impact derived from several kinds of disturbances. In this sense, a central conclusion of Obstfeld and Rogoff’s model is that, in the presence of monopolistic distortions, small expansionary policies (no matter where they originate) have similar effects on national welfare levels through their impact on global consumption. But in a global setting with vertically
integrated trade, can we continue to maintain this result? In other words, is it possible to rule out the presence of the “beggar-thy-neighbor” or “beggar-thyself” phenomenon? Here we show that in our setup the above result of the Redux model is reversed in such a way that a permanent relative monetary shock always gives rise to asymmetric effects on welfare in the two countries, leading to “beggar-thyself” or “beggar-thy-neighbor” problems.

Following the Redux model, in order to evaluate the welfare effects in detail, we write the utility function divided into two parts as \( U = U^R + U^M \), where \( U^R \) denotes the terms depending on output and consumption and \( U^M \) represents the term which depends on real money balances. In evaluating welfare, we must consider the following aspects. First, since any permanent monetary expansion has, as we have previously seen, effects on both short-run and long-run values of consumption, real money balances and output, both effects must be aggregated according to the weights implied by the utility function to obtain the total welfare effect. Second, as in the Redux model, we will omit the direct welfare impact of real balances since this effect seems to be empirically small.

Hence, from the utility functions for home and foreign agents (6), we may write the foreign and home utility changes as

\[
\begin{align*}
\frac{dU}{d\varepsilon} &= c - k\bar{Y}^*\varepsilon y + \beta(1 - \beta)^{-1}(\bar{c} - k\bar{Y}^*\varepsilon y), \\
\frac{dU^*}{d\varepsilon} &= c^* - k\bar{Y}_0^*\varepsilon y* + \beta(1 - \beta)^{-1}(\bar{c}^* - k\bar{Y}_0^*\varepsilon y*).
\end{align*}
\]  

(30)

In what follows, without loss of generality, we will consider a relative home monetary expansion given by \( \bar{m} > 0 \) and \( \bar{m}^* = 0 \). Also, we examine the welfare effects assuming that the consumption elasticity of money demand is the unity. Then, by using the results of the previous section, we can obtain the effect of that monetary shock on home and foreign welfare as functions of changes in the exchange rate in response to the monetary shock:\(^{19}\)

\[
\begin{align*}
dU^R &= -\Pi e, \quad dU^R* = \Pi^* e, 
\end{align*}
\]  

(31)

where

\[
\Pi = \left\{1 + \Phi + (1 - D^2)[2\Phi(\Phi - 1) - 1](\Phi)^{-1}\right\}/(1 - D^2 + \Phi) > 0,
\]

\[
\Pi^* = (1 - D^2)\left\{2\Phi - 1 + D^2(\Phi - 1)[2\Phi(\Phi - 1) - 1]\Phi^{-1}\right\}/2(1 - D^2 + r) > 0.
\]

In (31) we can see that exchange rate appreciation (depreciation) raises (reduces) the home utility whereas it reduces (raises) the foreign utility. Therefore, monetary policy always becomes a “beggar-thy-neighbor” or “beggar-thyself” instrument in an environment of vertical specialization in trade.

In order to see the intuition behind the above result, we focus on how consumption and outputs are affected by home monetary expansion. As regards consumption, the adjustment process underlying the result is based on both real interest-rate and terms-of-trade movements: real interest-rate changes induce households to make intertemporal changes in their consumption. Thus, under a fall (rise) in the real interest rate, households tend to anticipate (to translate) their consumption from (to) the future. The strength of this channel depends on the intertemporal elasticity of substitution which is assumed to be one in this paper, as in the Redux model. The second mechanism reflects the effect of real revenue changes associated to terms-of-trade movement.
It is in an open economy where consumption and output can differ due to changes in the terms of trade. In our setup of full vertical integration in trade, residents in each country will increase their relative real revenue and consumption when their terms of trade improve. But the direction of real interest-rate and terms-of-trade changes depends crucially on the direction of impact of the monetary shock on the exchange rate. We will need, therefore, to separate the depreciation and appreciation cases. On the other hand, within the effect on output we must distinguish between short and long run. While in the short run output becomes determined by world demand (for small enough shocks), in the long run output is derived from both terms-of-trade movements and the optimal tradeoff between output (effort) and leisure. The formal expressions for consumption and output changes are shown in the Appendix, so here we emphasize the intuition.

We start by illustrating the welfare effects under a depreciation scenario. In this case, a domestic monetary expansion decreases the home consumption in both the short and long run due to an increase in the real interest rate and a worsening of the terms of trade. Output generates opposite effects in the short run and long run. In the short run, since world demand falls, home output must decline. However, in the long run, at the margin, the utility cost of lost leisure is equal to the benefit from consumption financed by the income generated by supplying additional output. Hence, output supply will be higher (other things being equal) when consumption is lower. Then, consumption and output changes tend to cause a decline in the home long-run utility. Although in the short run a fall in both consumption and output has conflicting effects on home utility, it is easy to see that the welfare loss from a lower level of domestic consumption dominates the welfare benefit from a declining disutility of effort. Hence, when a relative home monetary expansion leads to a depreciation of the home currency, we find that $dU^R < 0$ and the monetary shock becomes unambiguously a “beggar-thyself” problem.

At the same time, in (31) we can see that the depreciation of the home currency reverses this finding in the foreign economy, causing the monetary shock to generate a positive international welfare effect. Indeed, in the short run, the improvement of the terms of trade offsets the effect of the rise in the real interest rate, leaving foreign consumption unchanged, and accordingly the fall in output (hence, effort) leads to a rise in utility. In the long run, although the rise in both consumption and output acts in opposite directions on utility (consumption raising it and output causing it to decline), the net effect is positive. Therefore, overall foreign welfare is boosted ($dU^{FE} > 0$), following a depreciation, by a home monetary expansion.

Consider, finally, the effect of a home monetary expansion on welfare when its impact on exchange rate is an appreciation. We must note that now the real interest rate falls, which raises the impact of a monetary shock on current consumption, and the terms of trade improve the purchasing power of domestic agents, leading to higher consumption expenditure. Thus, the home short-run welfare is subject to two contrasting forces: while a higher consumption tends to increase utility, greater output and effort tends to reduce it. However, the welfare benefit from a higher level of consumption dominates the disutility of the additional work effort. In the long run, home consumption increases because the revenue of home households is higher (due to an improvement of the terms of trade), inducing smooth consumption through a current account surplus. Hence, output must be reduced to optimize the effort–leisure tradeoff. Therefore, such changes in consumption and output reinforce each other, increasing the long-run welfare impact. So, by combining both short- and long-run effects, we conclude that overall home welfare rises ($dU^{RE} > 0$) when a home monetary
expansion generates a home currency appreciation, such as can be observed in equation (31).

By a similar mechanism to that previously outlined, one could demonstrate that the welfare impact on a foreign country of an increase in home money supply is unambiguously negative \((dU^R < 0)\), generating a kind of “beggar-thy-neighbor” phenomenon not derived from a competitive devaluation but from what could be called an “income switching effect.”

6. A Quantitative Illustration

We briefly show a numerical illustration of the model in order to assess the sensitivity of the main results to various values of the structural parameters of the economy. To do this, we set the values of the consumption elasticity of demand for money \((1/e)\) and the parameter \(\beta\) equal to one (as in the previous section) and 0.94 (Betts and Devereux, 2000) respectively.\(^{24}\) In this framework, for various values of the parameters \(\vartheta\) and \(D\) we evaluate, following Tille (2001), the impact of a unit monetary expansion in the home country on home and foreign welfare.

Table 1 shows the welfare effects, in absolute changes, in both countries \((dU^R\) and \(dU^{R*}\)) for various combinations of values of \(\vartheta\) and \(D\). Also, we present the results for various shares of intermediates in the value of final goods \((sh)\).

In our setup, parameter \(\vartheta\) plays a triple role: as elasticity of substitution between goods, as index of monopolist distortion, and as determinant of the share of intermediate goods value in the final goods value. Thus, as a degree of monopolist distortion, it can be derived from the empirical estimates of markups. There are various estimates on markups for the US economy. Rotemberg and Woodford (1992) argue that there is an average markup of 1.2, which implies an elasticity parameter \(\vartheta\) equal to 6. Nevertheless, Basu and Fernald (1994) suggest a lower markup of about 1.1, and hence the value of \(\vartheta\) would be of 11. On the other hand, since the share of the intermediate good in final output is given by \(sh = (\tilde{x}/D) = (1 - D^2)/[\vartheta(\vartheta - 1)^{-1} - D^2]\), the key point here is what values of \(D\) and \(\vartheta\) are consistent with each \(sh\) assumed. Hence, high values of \(\vartheta\) correspond to high shares of intermediates in the value of final output.\(^{25}\) Table 1 shows that only for values of \(\vartheta\) equal to (or greater than) 18, the home (foreign) welfare improves (worsens).\(^{26}\) As is also shown in that table, for the given weights of intermediates in the final-good production technology, this value of \(\vartheta\) corresponds to shares \((sh)\) equal to or greater than 0.7. Basu (1995) finds that an empirical plausible range

<table>
<thead>
<tr>
<th>(\vartheta)</th>
<th>(D)</th>
<th>(sh)</th>
<th>(dU^R)</th>
<th>(dU^{R*})</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.73</td>
<td>0.7</td>
<td>-3.1</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>0.5</td>
<td>-6.1</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>0.93</td>
<td>0.4</td>
<td>-11.1</td>
<td>7.6</td>
</tr>
<tr>
<td>11</td>
<td>0.73</td>
<td>0.8</td>
<td>-5.6</td>
<td>26.3</td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>0.7</td>
<td>-10.9</td>
<td>26.4</td>
</tr>
<tr>
<td></td>
<td>0.93</td>
<td>0.5</td>
<td>-19.6</td>
<td>26.4</td>
</tr>
<tr>
<td>18</td>
<td>0.73</td>
<td>0.9</td>
<td>114.9</td>
<td>-888.0</td>
</tr>
<tr>
<td></td>
<td>0.87</td>
<td>0.8</td>
<td>219.4</td>
<td>-893.5</td>
</tr>
<tr>
<td></td>
<td>0.93</td>
<td>0.7</td>
<td>393.7</td>
<td>-896.2</td>
</tr>
</tbody>
</table>
of share values is between 0.8 and 0.9. The notion that intermediate goods represent such a high proportion of the value of gross output may seem unrealistic; but as Gallup et al. (1998) have pointed out, such is the case for many key export sectors in developing countries, which import a very high proportion of the value of final output. In fact, in order to assess the impact of transport costs on the growth rate of two vertically integrated economies, they suppose $sh = 0.7$. Therefore, in accordance with the simulation results which are shown in Table 1, the larger the share of intermediate goods in final demand, the greater is the likelihood that monetary policy becomes a “beggar-thy-neighbor” instrument.\(^{27}\) In Table 1 the minimal share threshold for this effect is 0.7. So, from this share threshold, monetary policy can generate a “beggar-thy-neighbor” or “beggar-thyself” phenomenon depending on whether the value of parameter $\theta$ is high enough or not; that is, if the monetary shock brings about a depreciation or an appreciation of the home currency. Instead, for any share lower than 0.7 (for $\theta$ equal to 6 and 11), monetary policy always induces a “beggar-thyself” problem.\(^{28}\)

7. Concluding Remarks

This paper has presented an application of the Redux model of Obstfeld and Rogoff to analyze the impact of a monetary shock in the context of North–South integrated trade flows (intermediate goods in the North “versus” final goods in the South) on—among other variables—exchange rates and welfare. As such, it extends the contribution of Obstfeld and Rogoff to a field which has proved to be a standard workhorse in international trade theory. In this new framework, we have found that the impact of a monetary shock is very different from that of the Redux model. Indeed, first, a Northern monetary expansion might lead to an appreciation or depreciation of its currency depending on the values of the structural parameters of the model, such as consumption elasticity of money demand and monopoly power of producers. Specifically, we have found the surprising result of an appreciation in Northern currency if the values adopted by consumption elasticity of money demand are large enough and/or the degree of monopolist distortion is low enough. Also, if the consumption elasticity of money demand is not “too” high and/or the degree of monopoly is not “too” low, a relative home monetary expansion gives rise to a rise in the exchange rate, but, unlike in the Redux model, it is greater than the increase in the relative home money supply. Second, in our North–South trade setup, while the Northern monetary shock generates comovements in North and South outputs, the impact on welfare is, however, asymmetric since a “beggar-thy-neighbor” or “beggar-thyself” problem always arises depending, respectively, on whether the home currency appreciates or depreciates in the wake of a home monetary shock.

Finally, we should emphasize that the analysis of this paper can be extended to a number of dimensions. Here we will outline just some of them. An obvious extension of this paper would be to analyze fiscal policy in this framework and, also, another option would be to allow for the effects of different productivity shocks across regions (North and South). It would still be interesting to apply the analysis of this paper to the currency-related question of pricing intermediates and final goods. Thus, instead of assuming that export prices are set in the producer’s currency, we could consider that prices of both intermediates and final goods are set in Northern currency, or else that they are set in each country in terms of the local currency (“pricing-to-market”). In the first case, where all export prices are set in the North’s currency, the analysis would now show that the home currency depreciates unambiguously whereas both countries gain from an unexpected home monetary expansion. Therefore, in such a situation both
"beggar-thy-neighbor" or "beggar-thyself" would be excluded. However, under the "pricing-to-market" setting, one could prove that a home monetary expansion could lead to the "beggar-thy-neighbor" phenomenon since it raises the home utility whereas foreign welfare could decline.

Appendix

Comparing Steady States

The linearization of equations referring to the optimal tradeoff between labor (output) and leisure (equations (13) and (14)) yields

\[(\theta + 1)y = \theta(s - c) + y^*, \quad (A1)\]

\[k\bar{Y}_0^\varphi[(1 - (\bar{S}_o/D))(c + y^*) + (\theta - 1)\theta^{-2}(y^* - c^w) + (\bar{S}_0/D)s] = 0.\]

In a similar way, using the same linearization method, we can express the steady-state consumption equations (15) as

\[\bar{c} = (D\bar{r}/2\bar{S}_o\bar{b}) + \bar{y} + \bar{s}; \quad \bar{c}^* = -[1 - (\bar{S}_o/D)]^{-1}[(\bar{r}\bar{b}/2) + (\bar{S}_0/D)\bar{s}] + \bar{y}^*. \quad (A2)\]

From (A1) and (A2) together with equation (19) in the text, we obtain the expressions for steady-state consumption and terms-of-trade changes as a function of foreign assets changes (equations (20) and (21)).

Current Account

As, in the short run, nominal rigidity of prices prevails, we have that

\[p_t = p_t(z) = 0; \quad p_t^* = -e; \quad p^* = p^*(z^*) = 0; \quad p = e. \quad (A3)\]

Then the linearized short-run current-account equations for home and foreign country, respectively, are

\[\bar{b} = (2\bar{S}_o/D)(y - e - c); \quad \bar{b}^* = 2[1 - (\bar{S}_o/D)](y^* - c^*) + 2(\bar{S}_0/D)e. \quad (A4)\]

From (A4), by making use of the identity of net world foreign assets must be zero \((\bar{b} + \bar{b}^* = 0)\), we derive equation (22):

\[\bar{b} = -(2\bar{S}_o/D)[1 - (\bar{S}_0/D)](c - c^*) + e. \quad (A5)\]

Long-run and Short-run Changes

From (20) and (28), since we can express world consumption as \(c^w = (\bar{S}_o/D)c + [1 - (\bar{S}_o/D)]c^*,\) using equations (16), (18), and (29) we obtain

\[\bar{c} = -\frac{(1 + D^2)\bar{r}}{2(1 - D^2)\Delta} \frac{\bar{m}}{m}; \quad \bar{c}^* = \frac{(2\theta - 1)\bar{r}}{2\Delta} \frac{\bar{m}}{m}; \quad \bar{c}^w = \frac{\bar{r}}{2\Delta} \frac{\bar{m}}{m}; \quad (A6)\]

where \(\Delta = [1 - (\bar{r}(\theta - 1))].\)

From equations of demand for money, (12) in the text, we derive that

\[\bar{m} - e = c - \beta(\bar{r}^{-1}d + (1 - \beta)^{-1}(\bar{p} - e)); \quad 0 = c^* - \beta(\bar{r}^{-1}d + (1 - \beta)^{-1}\bar{p}^*); \quad (A7)\]
Moreover, using the Euler equations in the text, equations (11), we can write

\[ \bar{c} = \bar{c} - (1 - \beta) \bar{r}^{-1} dr; \quad \bar{c}^* = \bar{c}^* - (1 - \beta) \bar{r}^{-1} dr; \quad \bar{c}^w = \bar{c}^w - (1 - \beta) \bar{r}^{-1} dr. \]  

(A9)

Hence, from (A6)–(A9), we obtain

\[ \frac{dr}{\bar{r}} = \frac{(2 \theta - 1)}{2 \beta \Delta} \frac{\bar{m}}{\bar{m}}; \quad c = \frac{[\theta - D^2(\theta - 1)] \bar{r}}{(1 - D^2) \Delta \bar{m}}; \quad c^w = 0; \quad c^w = \frac{[\theta - 1]}{\Delta} \bar{m}. \]  

(A10)

Welfare Effects

Using (25) together with (A6) and (A10) to substitute in equations (30), we arrive at (31), which are the formal expression of welfare effects in response to exchange rate changes.

References


Notes
1. See, for example, Rose (1991) and Baier and Bergstrand (2001).
2. Balassa (1967) and Findlay (1984) were apparently the first to note this phenomenon in international trade, in calling it vertical specialization.
3. For these empirical evidences, see, for instance, Hummels et al. (1998) and Hummels et al. (2001). The former contains, primarily, evidence of cases of vertical integration such as, among others, US trade with the Mexican *maquiladoras*, and the Japanese electronics trade (that is, North–South vertical specialization). The latter, using input–output tables from 10 OECD and four emerging countries, studies the nature and growth of vertical specialization in world trade between 1970 and 1990.
4. However, our approach would also be extendable to either geographic orientation of vertical specialization.
5. The North–South model term was used by Findlay (1984) to refer to any model in which some basic asymmetry related to the stage of development between the two regions or countries is assumed.
6. Ruled out by assumption in this simple model is the notion that the “middle” stages of the productive spectrum might be “thick” in the sense that tradable middle goods might use other tradable middle goods as inputs.
7. Also, our setup could readily be used to allow for other patterns of trade. For instance, the pattern of trade where the South exports “primary products,” such as oil or raw materials, which are used in the North’s production process to produce final goods exported to the world markets.
8. In the data, cross-country correlations of output are typically higher than cross-country consumption correlations. Backus et al. (1992) refer to this as “quantity anomaly.”
9. In Lane (2001) we can find several directions in which the *Redux* model has been extended.
10. Although in a more general formulation the sizes of home and foreign countries are, respectively, \( n \) and \( 1 - n \), we do not need, here, to introduce possible differences in country size \( n \) because it is not essential for any of the central points made below.
11. A more general production technology than equation (3) would allow the elasticity of substitution between labor and intermediate goods, \( \sigma \), to differ from zero. This would be the case of, for instance, production technology CES:

\[
Y^*(z^*) = \left( \delta(L)^{(1-\sigma)} \right) + (1-\delta)(I)^{(1-\sigma)} \right]^{1/(1-\sigma)}, \quad 0 < \delta < 1.
\]
Considering this more general production technology of course enriches the analytical results of the model but, for empirically reasonable values of $\sigma$, the central results would not be qualitatively modified. In fact, one could prove that for all $\sigma < 1$, the findings would be similar to that of Leontief's technology. In a vertical specialization framework where labor is used in assembly operations, the substitutability between labor and intermediates tends to be very low. Thus, to mention just some case of vertical specialization, Truett and Truett (2001) find that labor and intermediates are complements in the Spanish automotive industry.

12. Here we adopt a weight $D$ in the final goods production between 0 and 1 (that is, more than one unit of intermediate is required to produce one unit of final goods). Doing this, given identical size of countries as well as equal elasticities of substitution for both final goods and intermediates, we ensure monopolist distortions in both intermediate and final goods markets as well as a positive price level. However, the model could also be reworked for other (either) values of $D$ without altering the central results of the paper.

13. As we only focus on monetary shocks, we assume that there is no government spending, so Ricardian equivalence adopts the form $0 = T_t + (M_t - M_{t-1})P_t^{-1}$.

14. It is easy to show that the change in relative home real income is $-\theta(1 - D^2) + D^2 e$.

15. This means that exchange rate overshooting is not possible here either.

16. Hence, even in the case of depreciation, the result in our setup is different from that of the Redux model, where the magnitude of the depreciation is always smaller than the increase in the relative money supply. However, it would be similar in a certain sense to other Redux model extensions such as those developed by Betts and Devereux (2000), and Tille (2001). But, the greater volatility in the works of Betts–Devereux and Tille is based on a PTM approach and the lack of substitutability between home and foreign goods, respectively, and in our setup the source of that volatility is in the idea of vertical specialization in trade, that is, when countries specialize only in a particular stage of a good’s production chain.

17. These results are obtained taking into account (16), (18), (25), and (26) to substitute in (20) and (22).

18. Empirical estimates of this elasticity are around unity or below, that is, for $\varepsilon \geq 1$ (Mankiw and Summers, 1986; Helliwell et al., 1990).

19. We relegate the derivation of equation (31) to the Appendix.

20. The effect on the real interest rate is derived in the Appendix.

21. We can express domestic consumption changes as $c = y + [1 - (S_0/D)](c - c*)$. This implies that the consumption impact is greater than the output impact.

22. In the foreign country, the price of intermediates would reduce relative to final goods prices, which would lead to a rise in the marginal utility of the additional revenue earned from producing an extra unit of final good. Hence the foreign producers would raise their effort and output. However, taking into account that $c^* = \bar{c} - (\bar{S}/D) \bar{c}^* - c^*$, the net effect must be positive.

23. The argument is similar to that of note 21.

24. This value of $\beta$ determines a steady-state real interest rate, $(1 - \beta)\beta^{-1}$, of 6%.

25. It is obvious that $(\partial sh/\partial \theta) > 0$.

26. This is the case in which a home monetary expansion brings about an appreciation of the home currency.

27. This is because the higher the share (the higher the value of $\theta$), the greater is the possibility of the home currency appreciating.

28. As we have seen in section 3, this was the condition required for a depreciation of the home currency.